Hadronic Part of the Muon $g-2$ Estimated on the $\sigma^{2003}_{\text{total}}(e^+e^- \to hadrons)$ Evaluated Data Compilation

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Abstract

A comprehensive as of November 2003 and evaluated data compilation on $\sigma_{tot}(e^+e^- \to hadrons)$ was used to estimate the lowest order hadronic contribution to the muon anomalous magnetic moment. The preliminary result is

$$a_\mu(\text{had}, \text{LO}) = (699.6 \pm 1.9_{\text{rad}} \pm 2.0_{\text{proc}} \pm 8.5_{e^+e^-}) \times 10^{-10}$$

The Standard Model value of the muon magnetic anomaly calculated by updated SM formulae published or e-printed by November 2003 then reads

$$a_\mu = 1.165\,918\,35(87_{\text{had}})(40_{\text{LbL}})(03_{\text{QED}})(02_{\text{EW}}) \times 10^{-3}$$

Introduction

Recent progress in refining the experimental and theoretical knowledge on the muon magnetic moment anomaly, which is one of the most sensitive to possible new effects in particle physics (beyond the Standard Model) quantities, is summarized in the Table 1.
It is seen that experiment is already stable and is evolving to the more accurate value whereas the theoretical estimates, in spite of intense activity, are far from being stable.

The bottleneck in the theoretical evaluation of $g-2$ is the hadronic contributions, especially the lowest order one. It cannot be found within the perturbative approach.

The problem is traditionally treated phenomenologically: the imaginary part of the hadronic vacuum polarization operator Π^{had}, through the real analyticity and asymptotic boundedness property can be related to the total cross section of the process $e^+e^- \to hadrons$ and the Π^{had} can be reconstructed from the experimental data using a dispersion relation technique.

This requires on the one hand a complete (to date) and accurate database of evaluated data on $\sigma_{tot}(e^+e^- \to hadrons)$ extracted from the original publications, and on the other

Table 1: In the column "Experiment" the 1998 entry is the CODATA recommended value, 1999-2002 entries are the world averaged values quoted in the cited experimental papers. In the column "Theory" the successive theoretical values calculated in the SM framework are presented. Asterisk marks the corrected result for the second 1998 and the 1999 theoretical entries.

Year	Experiment (BNL-821 et al.)		Theoretical estimates in SM		
1998	$1.165\,916\,02(64) \times 10^{-3}$	[1]	$1.165\,916\,45(156) \times 10^{-3}$		[2]
			$1.165\,916\,87(96) \times 10^{-3}$		[3]
1999	$1.165\,923\,5(73) \times 10^{-3}$	[5]	$1.165\,916\,3(8) \times 10^{-3}$		[4]
2000	$1.165\,920\,5(46) \times 10^{-3}$	[6]	$1.165\,915\,97(67) \times 10^{-3}$		[7]
2001	$1.165\,920\,3(15) \times 10^{-3}$	[8]	$1.165\,918\,49(69) \times 10^{-3}$		[9]
			$1.165\,918\,56(96) \times 10^{-3}$	*	[10]
2002	$1.165\,920\,3(8) \times 10^{-3}$	[11]	$1.165\,916\,93(70)(35)(04) \times 10^{-3}$		[12]

hand a stable (traceable) and reproducible in further refinements method of integration of the experimental data.

The Standard Model value of the muon anomalous magnetic moment can be conventionally broken into following parts:

$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{had}) \tag{1}$$

with

$$a_\mu(\mathrm{had}) = a_\mu(\mathrm{had, LO}) + a_\mu(\mathrm{had, HO}) + a_\mu(\mathrm{had, LbL})\ . \tag{2}$$

In the breakdown of the hadronic contribution (2) the terms are as follows:

- $a_\mu(\mathrm{QED}) = (11\,658\,470.6 \pm 0.3) \times 10^{-10}$ arises from the diagrams including only photon and charged lepton lines and calculated up to four loops (see a recent review [20] and references therein)
- $a_\mu(\mathrm{EW}) = (15.4 \pm 0.1_{\text{hadronic loops}} \pm 0.2_{M_{\text{Higgs}}}) \times 10^{-10}$ arises from one- and two-loop diagrams with W, Z and Higgs internal lines [21];
- $a_\mu(\mathrm{had})$ includes: the lowest order hadronic contribution (Fig. 1a) of a typical size

  $$a_\mu(\mathrm{had, LO}) \simeq 700 \times 10^{-10};$$

  higher order hadronic contributions (Fig. 1b) [22]

  $$a_\mu(\mathrm{had, HO}) = (-10.1 \pm 0.6) \times 10^{-10};$$

  light-by-light scattering contribution (Fig. 1c) ([28] and references therein)

  $$a_\mu(\mathrm{had, LbL}) = (8 \pm 4) \times 10^{-10}.$$

Current situation in theoretical estimates of the muon anomaly is illustrated in the Table 2.

Table 2: Recent SM calculations using different methods and different strategies in treatment of experimental data on $\sigma_{tot}(e^+e^- \to hadrons)$

Year	Theoretical estimates in SM	
2002	$1.165\,916\,93(70_{\rm had})(35_{\rm LbL})(04_{{\rm QED}+EW}) \times 10^{-3}$	[12]
2002	$1.165\,918\,89(78) \times 10^{-3}$	[13]
2002	$1.165\,916\,69(74) \times 10^{-3}$	[14]
2002	$1.165\,917\,26(77) \times 10^{-3}$	[14]
2003	$1.165\,916\,96(94) \times 10^{-3}$	[15]
2003	$1.165\,916\,75(75_{\rm had})(40_{\rm LbL})(04_{{\rm QED}+EW})(\times 10^{-3}$	[16]
2003	$1.165\,918\,12(127) \times 10^{-3}$	[17]
2003	$1.165\,918\,09(72_{\rm had})(35_{\rm LbL})(04_{{\rm QED}+EW}) \times 10^{-3}$	[18]
2003	$1.165\,918\,56(64_{\rm had})(35_{\rm LbL})(04_{{\rm QED}+EW}) \times 10^{-3}$	[19]
2003	$1.165\,918\,35(87_{\rm had})(40_{\rm LbL})(03_{\rm QED})(02_{\rm EW}) \times 10^{-3}$	[**this work**]

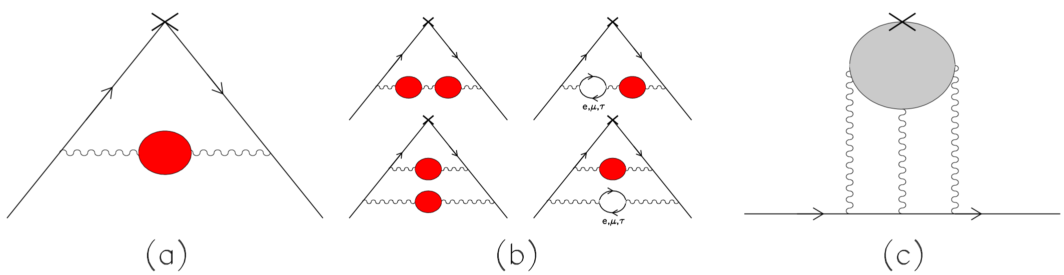

Figure 1: Lowest `(a)`, and higher order `(b),(c)` hadronic contributions to a_μ. Shaded and empty circles on the graphs `(a),(b)` denote hadronic and leptonic vacuum polarization operators, respectively. A shaded circle on the graph `(c)` denotes an effective four photon vertex dominated by the pion pole.

Details of the calculation

In this Letter we concentrate on the lowest order hadronic contribution to a_μ. An evaluation of the Fig. 1a diagram leads due to dispersion relation to a computationally convenient representation of $a_\mu(\text{had, LO})$ [23]:

$$a_\mu(\text{had},\text{LO}) = 4\alpha_0^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) \frac{1}{\pi} \,\text{Im}\,\Pi^{\text{had}}(s) = \frac{\alpha_0^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\text{had}}(s) \ , \qquad (3)$$

where

$$R^{\rm had}(s) = \sigma_{\rm tot}(e^+e^- \to hadrons, \text{lowest order in } \alpha) \bigg/ \frac{4\pi\alpha_0^2}{3s} \tag{4}$$

is the well known hadronic R-ratio, and the integration kernel is [23]

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)} \ . \tag{5}$$

An explicit expression for the kernel at $\sqrt{s} > 2m_\mu$ reads

$$K(s) = x^2\left(1-\frac{x^2}{2}\right) + (1+x)^2\left(1+\frac{1}{x^2}\right)\left(\ln(1+x) - x + \frac{x^2}{2}\right) + \frac{1+x}{1-x}x^2 \ln x \tag{6}$$

with $x = \left(1-\sqrt{1-4m_\mu^2/s}\right)\Big/\left(1+\sqrt{1-4m_\mu^2/s}\right)$, and at $\sqrt{s} < 2m_\mu$

$$\begin{aligned} K(s) \ = \ & \left[-4\left(\left(\frac{4m_\mu^2}{s}-1\right)^2 - 6\left(\frac{4m_\mu^2}{s}-1\right)+1\right)\arctan\sqrt{\frac{4m_\mu^2}{s}-1}\right. \\ & +\sqrt{\frac{4m_\mu^2}{s}-1}\left(\left(\frac{4m_\mu^2}{s}-1\right)^2 - (3+8\ln 2)\left(\frac{4m_\mu^2}{s}-1\right)\right) \\ & \left. +16\sqrt{\frac{4m_\mu^2}{s}-1}\left(\frac{2m_\mu^2}{s}-1\right)\ln\frac{4m_\mu^2}{s}\right] \Bigg/ \left[2\sqrt{\frac{4m_\mu^2}{s}-1}\left(\frac{4m_\mu^2}{s}\right)^2\right] \ . \end{aligned}$$

Experimental input

Experimental data on the total cross section of $e^+e^- \to hadrons$ were used to evaluate $R(s)$ in the range 0.36 GeV $< \sqrt{s} <$ $(12 \div 40)$ GeV. An indexed list of references to the publications related to our evaluated data compilation of the total hadronic cross sections used in our analysis is given in the Appendix and can be accessed in the machine readable form via `http://wwwppds.ihep.su:8001/eehadron.html`. We excluded all preliminary or withdrawn by their authors data. An overall view of the compilation is shown on the Fig. 2

Exclusive measurements.

At $\sqrt{s} < 2$ GeV the cross sections for exclusive hadronic final states were measured by Novosibirsk, Orsay and Frascati experiments (Tab. 3). Each measurement was rescaled first to the cross section in the improved Born approximation $\sigma^{\rm IBA}$, i.e. to a visible cross section corrected for ISR plus electronic vertex loops. The latter correction is always applied in the published data, therefore the only things left to us were to remove partial [24] or full vacuum polarization correction if applied by the data authors and account corrections for the photons radiated from the final state.

The partial correction for the electron vacuum polarization usually applied in earlier publications (before 1985) can be removed by factor

$$C^{\mathsf{b/m}}(s) \ = \ C_s^{\mathsf{b/m}}(s) \times C_{\rm Bhabha}^{\mathsf{b/m}}(s) \ ,$$

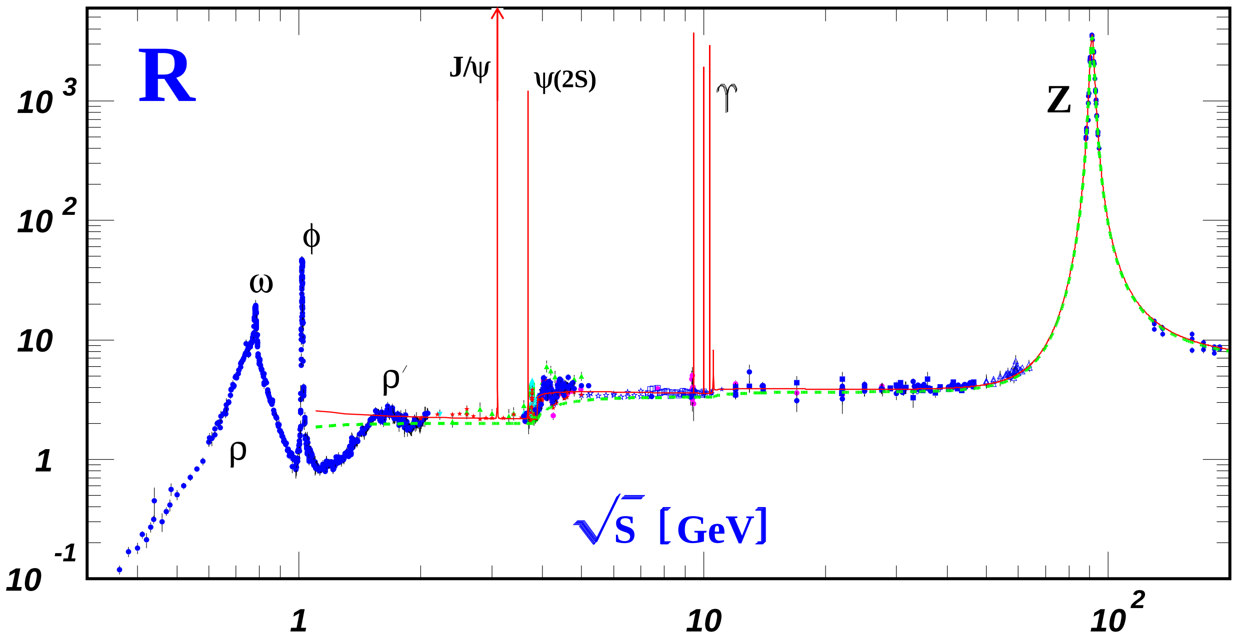

Figure 2: The compilation of data on $\sigma_{\rm tot}(e^+e^- \to hadrons)$ rescaled to the hadronic R ratio.

$$
\begin{aligned}
C_s^{\sf b/m}(s) &= \frac{1}{1-2{\rm Re}\Delta\alpha_e(s)}\ , \\
C_{\rm Bhabha}^{\sf b/m}(s) &= \frac{\int_{t_1}^{t_2} dt\ \sigma_{\rm Bhabha}\ (s,t)|_{\alpha(q^2)=\frac{\alpha_0}{1-\Delta\alpha(q^2)}}}{\int_{t_1}^{t_2} dt\ \sigma_{\rm Bhabha}\ (s,t)\,|_{\alpha(q^2)=\frac{\alpha_0}{1-\Delta\alpha_e(q^2)}}}
\end{aligned}
\tag{7}
$$

Here $C_s^{\sf b/m}(s)$ removes the correction for the electronic vacuum polarization in the s-channel and $C_{\rm Bhabha}^{\sf b/m}(s)$ properly corrects the large angle Bhabha scattering cross section used as a luminosity monitor in the low energy experiments. The squared momentum transfer range $[t_1, t_2]$ is individual for each detector [26]. Finally, $\Delta\alpha_e(s)$ and $\Delta\alpha(s)$ are related to the electronic and full vacuum polarization operators as

$$
\Delta\alpha_e(s) = \Pi_e(s)/s\ \ ,\ \ \Delta\alpha(s) = \Pi(s)/s\ , \tag{8}
$$

respectively. An expression for the one loop electronic (generally leptonic) vacuum polarization operator $\Pi_\ell(s)$ can be found elsewhere (see, e.g. [25]). The hadronic part of vacuum polarization cannot be calculated perturbatively and is related to the experimental hadronic R ratio as

$$
\Delta\alpha_{\rm had}(s) = -\frac{\alpha_0}{3\pi}\, s \int_{m_\pi^2}^{\infty} \frac{R(s')\, ds'}{s'(s'-s-i0)}\ . \tag{9}
$$

The full vacuum polarization correction applied by some later experiments is removed by the factor

$$
C^{\rm full}(s) = \frac{1}{1-2{\rm Re}\Delta\alpha(s)}\ , \tag{10}
$$

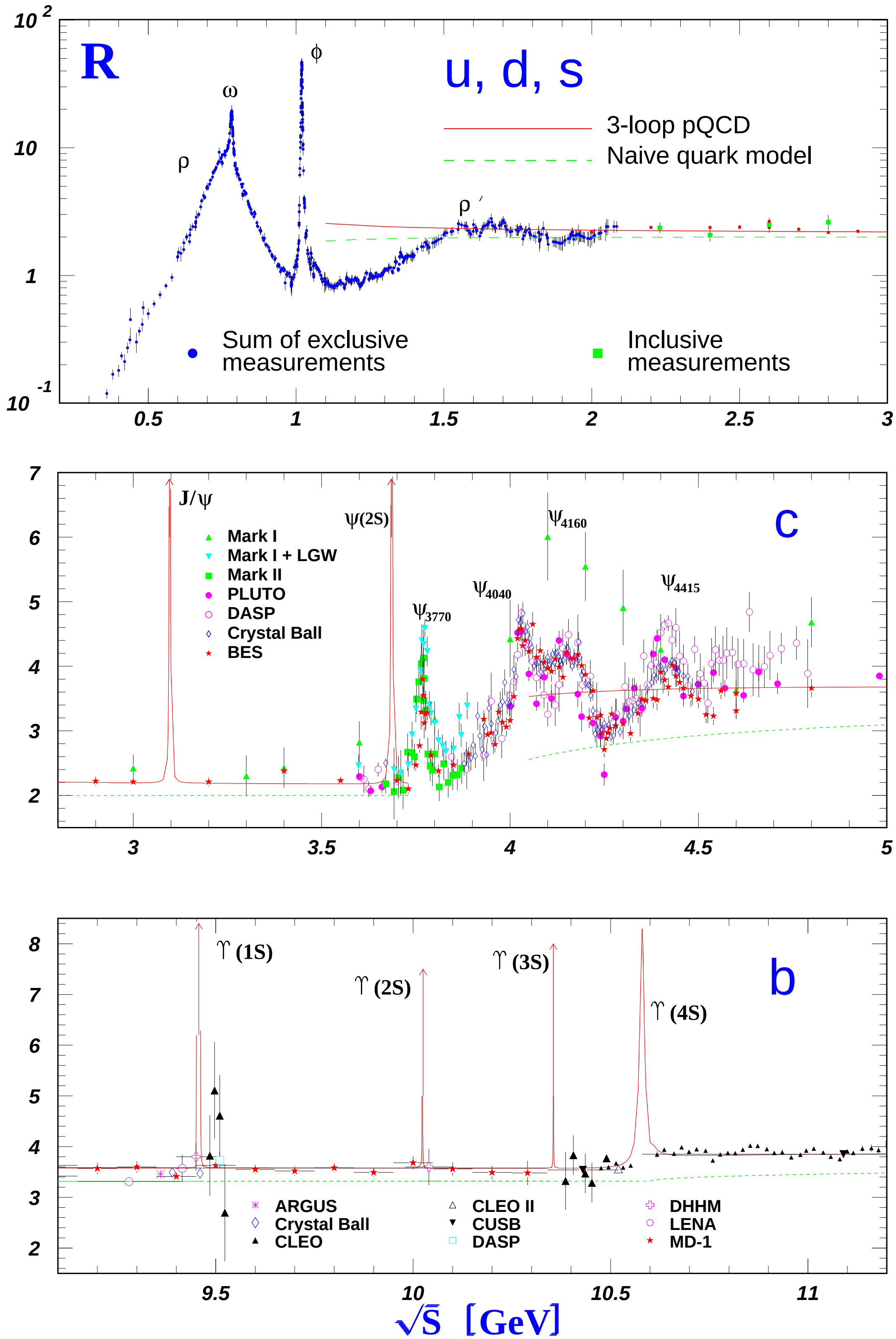

Figure 2: (continued) Threshold regions in the e^+e^- hadroproduction: u, d, s-, c- and b-flavour onset. Note the consistency between the exclusive and inclusive data at $\sqrt{2} \sim 2$ GeV.

where

$$\Delta\alpha(s) = \sum_{\ell=e,\mu,\tau} \Delta\alpha_\ell(s) + \Delta\alpha_{\rm had}(s) \tag{11}$$

with $\Delta\alpha_{\rm had}(s)$ calculated according to (9).

Turning off doubtful radiative corrections allows to estimate the "radiative" uncertainty of $a_\mu(\text{had}, \text{LO})$.

After rescaling the data to $\sigma^{\rm IBA}$ their weighed average $\overline{\sigma}^{\rm IBA}(s)$ for each exclusive hadronic channel is found by the inimization of the bilinear form

$$Q = \sum_{(k)} (C^{(k)})^{-1}_{ij} \left(\overline{\sigma}^{\rm IBA}(s_i^{(k)}) - \sigma^{\rm IBA}{}_i^{(k)}\right) \left(\overline{\sigma}^{\rm IBA}(s_j^{(k)}) - \sigma^{\rm IBA}{}_j^{(k)}\right) \ . \tag{12}$$

Here an index k runs over the experiments measured the given channel, indices i and j enumerate the data points within the k-th experiment and $C^{(k)}$ is the error matrix of the k-th experiment. We conservatively assumed no correlations between systematic uncertainties of data coming from different publications even if the measurements were performed at the same facility. Unfortunately, publications (especially older ones) do not provide enough information to split a total systematic uncertainty into separate sources which is necessary to find correlations between the experiments. Indeed, 100% correlation might be among uncertainties coming from a machine luminosity determination, radiative corrections, *etc.*, i.e. from the procedures common for all experiments. *Post factum* crude estimates [12] based on expert judgements are also questionable because the lack of descriptions of the systematic error sources can easily lead to double counting. These considerations justify our refusal (for the moment) to take into account the correlations between experiments despite a possible overestimate of the e^+e^- uncertainty of a_μ. This point requires further investigation.

Next, the averaged cross sections of each exclusive final state are summed up to give the total cross section $\sigma^{\rm IBA}_{\rm tot}(e^+e^- \to \textit{hadrons})$. The latter divided by $4\pi\alpha^2(s)/3s$ gives the desired hadronic ratio $R(s)$ entering the integral (3).

Summing the contributions of exclusive hadronic final states is not always straightforward. To account for the missing hadronic final states the isospin symmetry relations are exploited where possible (see the discussion in [12]).

Note that determination of $\Delta\alpha(s)$ in Eqs. (7), (10) in turn requires to evaluate the dispersion relation integral (9) also containing the R ratio. For this reason an iterative procedure was applied: at zero iteration $\Delta\alpha(s)$ was calculated in the naive approximation including one loop QED contributions from all quarks and charged leptons plus contributions from $\phi(1020)$, J/ψ, $\psi(2S)$ and $\Upsilon(1S)\ldots\Upsilon(4S)$ resonances; then the obtained naive $\Delta\alpha(s)$ was used to properly rescale the experimental data to the R ratio which in turn was used for the evaluation of $\Delta\alpha(s)$ to be utilized at the next iteration. This process proved to be well convergent even in the vicinities of narrow resonances which contribution to $\Delta\alpha(s)$ was obtained analytically as will be explained later. It turns out that two iterations are enough with the current level of accuracy.

$\pi^+\pi^-$, $\pi^0\gamma$ threshold regions.

The lowest experimental point of our compilation lies at $\sqrt{s} = 0.36$ GeV, well above $\pi^+\pi^-$ and $\pi^0\gamma$ production thresholds. The hadronic cross section in the $2m_\pi < \sqrt{s} < 0.36$ GeV range can be evaluated using the chiral perturbation theory (ChPT) parametrization of the pionic formfactor

$$F_\pi^{\rm ChPT}(s) = 1 + \frac{<r^2>_\pi}{6} s + c_1 s^2 + c_2 s^3 + \mathcal{O}(s^4) \; , \tag{13}$$

where the squared pionic charge radius $< r^2 >_\pi = (11.27 \pm 0.21)$ GeV^{-2} follows from the fit of space-like data [27] and the parameters c_1, c_2 are fitted to the time-like data in the range $2m_\pi < \sqrt{s} < 0.6$ GeV [12].

The two pion production cross section above the threshold then reads

$$\sigma_{\pi^+\pi^-} = \frac{\pi |\, \alpha(s)|^2}{3s} \, (1 - 4m_\pi^2/s)^{3/2} \, |F_\pi^{\rm ChPT}(s)|^2 \; . \tag{14}$$

The $\pi^0\gamma$ cross section in the range $m_\pi < \sqrt{s} < 0.6$ GeV is much smaller and parametrized using the $\pi^0 - \gamma^*\gamma$ transition formfactor [28]. The phenomenological parametrization of the low energy hadronic cross section can be reliably used up to $\sqrt{s_{\rm ChPT}} = 0.5$ GeV. The uncertainty due to the variation of $\sqrt{s_{\rm ChPT}}$ in the range 0.36 – 0.6 GeV is folded to the procedural error of $a_\mu(\rm had, LO)$.

Inclusive measurements.

At $\sqrt{s} \geq 2$ GeV the e^+e^- experiments mostly measure the total cross section of an inclusive production of hadrons (Tab. 4). Data published after 1978 seem to be fully corrected for ISR, electronic vertex loops and vacuum polarization. Earlier data with only electronic vacuum polarization correction need to be properly rescaled as mentioned above (Eq. 7). Data rescaled to the correct R ratio were weighed in the Eq. (12) manner to give the averaged R ratio entering into dispersion relation integrals (3) and (9).

Resonances.

The contributions of $\omega(782)$, $\phi(1020)$, $\psi(3770)$, $\psi(4040)$ and $\psi(4160)$ resonances are already contained in the cross section data as their widths are larger than a typical machine energy spread. Although, in this preliminary work we account for the contribution of a relatively broad $\phi(1020)$ meson using the relativistic Breit-Wigner parametrization

$$R_{res}(s) \; = \; \sigma_{\rm BW}(s) \; \Big/ \; \frac{4\pi|\alpha(s)|^2}{3s} \; = \; \frac{9}{|\alpha(s)|^2} \, \frac{s \, \Gamma_{ee} \Gamma \, \frac{s}{M^2}}{(s-M)^2 + s\, \Gamma^2} \; , \tag{15}$$

where Γ_{ee} and Γ are physical electronic and total widths of the resonance given by PDG [29]. Narrow J/ψ, $\psi(2S)$ and $\Upsilon(nS), n = 1..4$ resonances were treated in the same way.

Note that a dispersion relation in the form (9) should not be used to find the contribution of a narrow resonance to the running $\alpha(s)$ as the R ratio (15) in turn contains $\alpha(s)$ rapidly varying in the vicinity of a resonance. Instead, we use another form of the dispersion integral relating the values of $\alpha(s)$ with and without the contribution of the resonance:

$$\begin{aligned} \alpha(s) - \alpha_{\text{without}}(s) &= -\frac{s}{4\pi^2} \int \frac{\sigma_{\text{BW}}(s')\, ds'}{s' - s - i0} \\ &= -\frac{s}{\pi} \int \frac{3\,\Gamma_{ee}\Gamma\,\frac{s'}{M^2}}{(s'-M^2)^2 + s'\Gamma^2} \frac{ds'}{s' - s - i0} . \end{aligned} \tag{16}$$

This expression can be easily obtained from the analytic properties of the re-summed photon propagator.

High energy tail

The R ratio at $\sqrt{s} > 12$ GeV can be reliably evaluated using the perturbative QCD. We used a three loop pQCD approximation taking into account the effect of quark masses [30]. A variation of the lower boundary of pQCD usage in the range 12 – 40 GeV results in a negligible additional uncertainty of $a_\mu(\text{had}, \text{LO})$.

Discussion

To cross-check the obtained value of the lowest order hadronic contribution to the muon magnetic anomaly

$$a_\mu(\text{had}, \text{LO}) = (699.6 \pm 1.9_{\text{rad}} \pm 2.0_{\text{proc}} \pm 8.5_{e^+e^-}) \times 10^{-10} , \tag{17}$$

we repeated the procedure on the subset of our compilation of $\sigma_{\text{tot}}(e^+e^- \to hadrons)$ data used in recent papers [12, 18]. The result

$$a_\mu(\text{had}, \text{LO}, \text{subset}) = (694.5 \pm 1.9_{\text{rad}} \pm 2.0_{\text{proc}} \pm 8.8_{e^+e^-}) \times 10^{-10} , \tag{18}$$

is consistent with the corrected [18] e^+e^- based result of the paper [12]

$$a_\mu(\text{had}, \text{LO}, [18]) = (696.3 \pm 3.6_{\text{rad+proc}} \pm 6.2_{e^+e^-}) \times 10^{-10} . \tag{19}$$

The uncertainty induced by the e^+e^- data is larger in our work because of significant differences in the data treatment:

different integration procedures: we obtain the total R ratio first and then integrate it without averaging within small energy bins each including several experimental points; on the other hand, in [12, 18] the contributions of each hadronic final state were added separately and the aforementioned energy averaging was applied;

different treatment of systematic errors: no correlations between different experiments in this work versus significant correlations nominated in [12, 18].

These items will be clarified in the forthcoming publication.

Conclusion

A new estimate of the lowest order hadronic contribution to the muon anomalous magnetic moment was obtained using a comprehensive (as of November 2003) compilation of evaluated data on total hadronic cross sections in e^+e^- collisions. The preliminary result that is free of any extra admissions on the data not documented in the original experimental publications reads

$$a_\mu(\mathrm{had,LO}) = (699.6 \pm 1.9_{\mathrm{rad}} \pm 2.0_{\mathrm{proc}} \pm 8.5_{e^+e^-}) \times 10^{-10} ,$$

where the first error is due to the uncertainty in the radiative corrections to the e^+e^- data, the second one is procedural and the last one is due to the experimental errors of the e^+e^- data. The value of the muon magnetic anomaly then reads as

$$a_\mu = (11659183.5 \pm 8.7_{\mathrm{had}} \pm 4.0_{\mathrm{LbL}} \pm 0.3_{\mathrm{QED}} \pm 0.2_{\mathrm{EW}}) \times 10^{-10} ,$$

where the errors are from the hadronic, light-by-light scattering, pure QED and electroweak contributions, respectively. This result deviates from the experimental "world average" of a_μ [31] by

$$(-19.5 \pm 9.6_{\mathrm{theor}} \pm 8.0_{\mathrm{exp}}) \times 10^{-10} ,$$

i.e. at $\sim 1.5\sigma$ level.

As it can be seen from the Table 2 our estimate is well matched with all other 2002-2003 estimates based on the $e^+e^- \to hadrons$ data. The differences are due to slightly different databases used and different methods of incorporating experimental data into final estimates. It seems that to make a further progress in the refinement of these estimates it will be useful to standardize the database to meet all aspects of the scientific database quality: completeness, accuracy and traceability of the data transference from original publications to the evaluated data compilations, transparency of the data evaluation procedures, and easy access to the evaluated database in computer readable form for physics and education communities. Some steps towards such compilations were undertaken by the COMPAS and HEPDATA groups under auspices of the PDG collaboration [29, 30, 32].

The standardized and maintained evaluated data compilation will allow to join efforts and find most stable and reliable method of incorporating experimental data into the theoretical estimates of the hadronic contributions to the high precision observables and to trace the consistency of the different experimental evidences with the theoretical estimates in the Standard Model.

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Appendix

Column titles in Tabs. 3, 4 are self-explaining. R and σ in the "Obs." column denote the types of observables in the original papers: R ratio and cross section, respectively. The abbreviations in the last column of Tab. 3 and in the fifth column of Tab. 4 denote the types of radiative corrections (RC) applied in the original publications:

b/m – data corrected for the initial state radiation (ISR), e^+e^- vertex loops and electronic vacuum polarization [24];

full – data corrected for ISR, e^+e^- vertex loops and full vacuum polarization;

ISR – data corrected for ISR and e^+e^- vertex loops only.

Data not used in the calculations are marked by an asterisk (to compare, see the recent papers [12, 18, 32] where the extended experimental bibliography is also presented).

Table 3: Exclusive hadronic cross section measurements.

Experiment		Reference	Author *et al.*	Obs.	RC
		$\pi^+\pi^-(\gamma)$			
VEPP-2	*	Phys. Lett. **41B** (1972) 205	Balakin, V.E.	σ	b/m
VEPP-2M-OLYA		Nucl. Phys. **B256** (1985) 365	Barkov, L.M.	σ	b/m
VEPP-2M-TOF		Sov. J. Nucl. Phys. **33** (1980) 368	Vasserman, I.B.	σ	b/m
VEPP-2M-CMD		Nucl. Phys. **B256** (1985) 365	Barkov, L.M.	σ	b/m
ACO	*	Phys. Lett. **39B** (1972) 289	Benaksas, D.	σ	b/m
DCI-DM1		Phys. Lett. **76B** (1978) 512	Quenzer, A.	σ	b/m
DCI-DM2		Phys. Lett. **220B** (1989) 321	Bisello, D.	σ	full
ADONE-MEA		Nuovo Cim. Lett. **28** (1980) 337	Esposito, B.	σ	b/m
ADONE-BCF		Nuovo Cim. Lett. **15** (1976) 393	Bollini, D.	σ	b/m
VEPP-2M-CMD-2		Phys. Lett. **527B** (2002) 161	Akhmetshin, R.R.	σ	full
		Erratum* hep-ex/0308008 (2003)	Akhmetshin, R.R.		
		$\pi^+\pi^-\pi^0$			
VEPP-2M-CMD-2		Phys. Lett. **476B** (2000) 33	Akhmetshin, R.R.	σ	ISR
		Erratum* hep-ex/0308008 (2003)	Akhmetshin, R.R.		
		Phys. Lett. **364B** (1995) 199	Akhmetshin, R.R.	σ	ISR
		Phys. Lett. **434B** (1998) 426	Akhmetshin, R.R.	σ	ISR
VEPP-2M-SND		Phys. Rev. **D66** (2002) 032001	Achasov, M.N.	σ	ISR
		Phys. Rev. **D63** (2001) 072002	Achasov, M.N.	σ	ISR
		Phys. Rev. **D68** (2003) 052006	Achasov, M.N.	σ	ISR
VEPP-2M-ND		Phys. Rep. **202** (1991) 99	Dolinsky, S.I.	σ	ISR
VEPP-2M-CMD		Novosibirsk 89-15 (1989)	Barkov, L.M.	σ	ISR
DCI-DM1		Nucl. Phys. **B172** (1980) 13	Cordier, A.	σ	ISR
DCI-DM2		Z. Phys. **C56** (1992) 15	Antonelli, A.	σ	ISR
ACO	*	Phys. Lett. **28B** (1968) 513	Augustin, J.E.	σ	b/m
	*	Phys. Lett. **42B** (1972) 507	Benaksas, D.	σ	b/m
ADONE-$\gamma\gamma$2		Nucl. Phys. **B184** (1981) 31	Bacci, C.	σ	b/m
ADONE-MEA		Nuovo Cim. Lett. **28** (1980) 195	Esposito, B.	σ	b/m

Table 3: (continued) Exclusive hadronic cross section measurements.

Experiment	Reference	Author *et al.*	Obs.	RC
		$\boldsymbol{\pi^0\gamma}$		
VEPP-2M-SND	Eur. Phys. J. **C12** (2000) 25	Achasov, M.N.	σ	ISR
	Phys. Lett. **559B** (2003) 171	Achasov, M.N.	σ	ISR
ACO	Phys. Lett. **63B** (1976) 352	Cosme, G.	σ	ISR
		$\boldsymbol{\eta\ \gamma}$		
VEPP-2M-CMD-2	Phys. Lett. **364B** (1995) 199	Akhmetshin, R.R.	σ	ISR
	Phys. Lett. **509B** (2001) 217	Akhmetshin, R.R.	σ	ISR
VEPP-2M-SND	Eur. Phys. J. **C12** (2000) 25	Achasov, M.N.	σ	ISR
ACO	Phys. Lett. **63B** (1976) 352	Cosme, G.	σ	ISR?
		$\boldsymbol{\omega < \pi^0\gamma > \pi^0}$		
VEPP-2M-SND	Phys. Lett. **486B** (2000) 29	Achasov, M.N.	σ	ISR
	Nucl. Phys. **B569** (2000) 158	Achasov, M.N.	σ	ISR
VEPP-2M-CMD-2	Phys. Lett. **562B** (2003) 173	Akhmetshin, R.R.	σ	ISR
VEPP-2M-ND	Phys. Lett. **174B** (1986) 453	Dolinsky, S.I.	σ	ISR
DCI-DM2	LAL-90-35 (1990)	Bisello, D.	σ	ISR
		$\boldsymbol{\pi^+\pi^-2\pi^0}$		
VEPP-2M-OLYA	Zh. Exp. Th. Fiz. Pisma **43** (1986) 497	Kurdadze, L.M.	σ	ISR
VEPP-2M-ND	Phys. Rep. **202** (1991) 99	Dolinsky, S.I.	σ	ISR
VEPP-2M-CMD-2	Phys. Lett. **466B** (1999) 392	Akhmetshin, R.R.	σ	ISR
VEPP-2M-SND	Budker INP 2001-34 (2001)	Achasov, M.N.	σ	ISR
DCI-M3N	Nucl. Phys. **B152** (1979) 215	Cosme, G.	σ	b/m
ADONE-$\boldsymbol{\gamma\gamma}$**2**	Nucl. Phys. **B184** (1981) 31	Bacci, C.	σ	b/m
ACO	Phys. Lett. **63B** (1976) 349	Cosme, G.	σ	b/m
DCI-DM2	LAL-90-35 (1990)	Bisello, D.	σ	ISR
ADONE-MEA	Nuovo Cim. Lett. **31** (1981) 445	Esposito, B.	σ	b/m

Table 3: (continued) Exclusive hadronic cross section measurements.

Experiment	Reference	Author *et al.*	Obs.	RC
		$\boldsymbol{2\pi^+2\pi^-}$		
VEPP-2M-OLYA	Zh. Exp. Th. Fiz. Pisma **47** (1988) 432	Kurdadze, L.M.	σ	ISR
VEPP-2M-CMD	Sov. J. Nucl. Phys. **47** (1988) 248	Barkov, L.M.	σ	ISR
VEPP-2M-CMD-2	Phys. Lett. **475B** (2000) 190	Akhmetshin, R.R.	σ	ISR
	Phys. Lett. **491B** (2000) 81	Akhmetshin, R.R.	σ	ISR
	Phys. Lett. **466B** (1999) 392	Akhmetshin, R.R.	σ	ISR
VEPP-2M-SND	Budker INP 2001-34 (2001)	Achasov, M.N.	σ	ISR
VEPP-2M-ND	Phys. Rep. **202** (1991) 99	Dolinsky, S.I.	σ	ISR
DCI-M3N	Nucl. Phys. **B152** (1979) 215	Cosme, G.	σ	b/m
DCI-DM1	Phys. Lett. **81B** (1979) 389	Cordier, A.	σ	b/m
	Phys. Lett. **109B** (1982) 129	Cordier, A.	σ	b/m
DCI-DM2	LAL-90-35 (1990)	Bisello, D.	σ	ISR
ADONE-MEA	Nuovo Cim. Lett. **28** (1980) 195	Esposito, B.	σ	b/m
ADONE-$\boldsymbol{\mu\pi}$	Nuovo Cim. **13A** (1973) 593	Grilli, M.	σ	b/m
ADONE-$\boldsymbol{\gamma\gamma 2}$	Phys. Lett. **95B** (1980) 139	Bacci, C.	σ	b/m
ACO	Phys. Lett. **63B** (1976) 349	Cosme, G.	σ	b/m
		$\boldsymbol{2\pi^+2\pi^-\pi^0}$		
VEPP-2M-CMD	Sov. J. Nucl. Phys. **47** (1988) 248	Barkov, L.M.	σ	ISR
DCI-M3N	Nucl. Phys. **B152** (1979) 215	Cosme, G.	σ	b/m
ADONE-$\boldsymbol{\gamma\gamma 2}$	Nucl. Phys. **B184** (1981) 31	Bacci, C.	σ	b/m
ADONE-MEA	Nuovo Cim. Lett. **31** (1981) 445	Esposito, B.	σ	b/m
ADONE-$\boldsymbol{\mu\pi}$	Nuovo Cim. **13A** (1973) 593	Grilli, M.	σ	b/m
		$\boldsymbol{\omega < \pi^+\pi^-\pi^0 > \pi^+\pi^-}$		
DCI-DM1	Phys. Lett. **106B** (1981) 155	Cordier, A.	σ	b/m
DCI-DM2	Z. Phys. **C56** (1992) 15	Antonelli, A.	σ	b/m
VEPP-2M-CMD-2	Phys. Lett. **489B** (2000) 125	Akhmetshin, R.R.	σ	ISR
		$\boldsymbol{\eta\ \pi^+\pi^-}$		
VEPP-2M-CMD-2	Phys. Lett. **489B** (2000) 125	Akhmetshin, R.R.	σ	ISR
VEPP-2M-ND	Phys. Rep. **202** (1991) 99	Dolinsky, S.I.	σ	ISR
		$\boldsymbol{\pi^+\pi^-3\pi^0}$		
DCI-M3N	Nucl. Phys. **B152** (1979) 215	Cosme, G.	σ	b/m
ADONE-MEA	Nuovo Cim. Lett. **25** (1979) 5	Esposito, B.	σ	b/m

Table 3: (continued) Exclusive hadronic cross section measurements.

Experiment	Reference	Author *et al.*	Obs.	RC
		$3\pi^+3\pi^-$		
VEPP-2M-CMD	Sov. J. Nucl. Phys. **47** (1988) 248	Barkov, L.M.	σ	ISR
DCI-DM1	Phys. Lett. **107B** (1981) 145	Bisello, D.	σ	b/m
DCI-DM2	Roma U. thesis (1986)	Schioppa, M.	σ	ISR
DCI-M3N	Nucl. Phys. **B152** (1979) 215	Cosme, G.	σ	b/m
ADONE-$\mu\pi$	Nuovo Cim. **13A** (1973) 593	Grilli, M.	σ	b/m
		$2\pi^+2\pi^0 2\pi^-$		
VEPP-2M-CMD	Sov. J. Nucl. Phys. **47** (1988) 248	Barkov, L.M.	σ	ISR
ADONE-MEA	Nuovo Cim. Lett. **31** (1981) 445	Esposito, B.	σ	b/m
DCI-M3N	Nucl. Phys. **B152** (1979) 215	Cosme, G.	σ	b/m
ADONE-$\gamma\gamma 2$	Nucl. Phys. **B184** (1981) 31	Bacci, C.	σ	b/m
ADONE-$\mu\pi$	Nuovo Cim. **13A** (1973) 593	Grilli, M.	σ	b/m
DCI-DM2	Roma U. thesis (1986)	Schioppa, M.	σ	ISR
		K^+K^-		
VEPP-2	Phys. Lett. **41B** (1972) 205	Balakin, V.E.	σ	b/m
VEPP-2M-OLYA	Phys. Lett. **107B** (1981) 297	Ivanov, P.M.	σ	ISR
ADONE-BCF	Phys. Lett. **46B** (1973) 261	Bernardini, M.	σ	b/m
ACO-DM1	LAL-80-35 (1980)	Grosdidier, G.	σ	b/m
	Phys. Lett. **99B** (1981) 257	Delcourt, B.	σ	b/m
DCI-DM2	Z. Phys. **C39** (1988) 13	Bisello, D.	σ	ISR
ADONE-MEA	Nuovo Cim. Lett. **28** (1980) 337	Esposito, B.	σ	b/m
VEPP-2M-CMD	Novosibirsk 83-85 (1983)	Anikin, G.V.	σ	ISR
VEPP-2M-CMD-2	Phys. Lett. **364B** (1995) 199	Akhmetshin, R.R.	σ	ISR
VEPP-2M-SND	Phys. Rev. **D63** (2001) 072002	Achasov, M.N.	σ	ISR

Table 3: (continued) Exclusive hadronic cross section measurements.

Experiment	Reference	Author *et al.*	Obs.	RC
		$K_S K_L$		
DCI-DM1	Phys. Lett. **99B** (1981) 261	Mane, F.	σ	b/m
VEPP-2M-OLYA	Zh. Exp. Th. Fiz. Pisma **36** (1982) 91	Ivanov, P.M.	σ	b/m
VEPP-2M-CMD	Novosibirsk 83-85 (1983)	Anikin, G.V.	σ	b/m
VEPP-2M-CMD-2	Phys. Lett. **364B** (1995) 199	Akhmetshin, R.R.	σ	ISR
	Phys. Lett. **551B** (2003) 27	Akhmetshin, R.R.	σ	ISR
	Phys. Lett. **466B** (1999) 385, Errata *ibid.* **508B** (2001) 217, hep-ex/0308008 (2003)	Akhmetshin, R.R.	σ	ISR
VEPP-2M-SND	Phys. Rev. **D63** (2001) 072002	Achasov, M.N.	σ	ISR
		$K^+K^-\pi^0$		
DCI-DM2	Nucl. Phys. Proc. Suppl. **21** (1991) 111	Bisello, D.	σ	ISR
	Z. Phys. **C52** (1991) 227	Bisello, D.	σ	ISR
		$K_S K^+\pi^- + K_S K^-\pi^+$		
DCI-DM1	Phys. Lett. **112B** (1982) 178	Mane, F.	σ	b/m
DCI-DM2	Z. Phys. **C52** (1991) 227	Bisello, D.	σ	ISR
		$K^+K^-\pi^+\pi^-$		
DCI-DM1	Phys. Lett. **110B** (1982) 335	Cordier, A.	σ	b/m
DCI-DM2	Nucl. Phys. Proc. Suppl. **21** (1991) 111	Bisello, D.	σ	ISR
		$K_S + X$		
DCI-DM1	LAL-82-46 (1982)	Mane, F.	σ	b/m
		$p\,\bar{p}$		
DCI-DM1	Phys. Lett. **86B** (1979) 395	Delcourt, B.	σ	b/m
DCI-DM2	Nucl. Phys. **B224** (1983) 379	Bisello, D.	σ	b/m
ADONE-FENICE	Nucl. Phys. **B517** (1998) 3	Antonelli, A.	σ	full
		$n\,\bar{n}$		
ADONE-FENICE	Nucl. Phys. **B517** (1998) 3	Antonelli, A.	σ	full

Table 4: Inclusive hadronic cross section measurements.

Experiment	Reference	Author *et al.*	Obs.	RC	E_{cm} [GeV]
* ADONE-MEA	Phys. Lett. **58B** (1975) 478	Bartoli, B.	σ	b/m	2.23
BEPC-BES	Phys. Rev. Lett. **88** (2002) 101802	Bai, J.Z.	R	full	2.0 - 4.8
BEPC-BES	Phys. Rev. Lett. **84** (2000) 594	Bai, J.Z.	R	full	2.6 - 5.0
* SPEAR-SMAG †	Phys. Rev. Lett. **34** (1975) 764	Augustin, J.E.	σ	b/m	2.4 - 5.0
* SPEAR-SMAG+LGW	Phys. Rev. Lett. **39** (1977) 526	Rapidis, P.A.	R	full	3.598 - 3.886
SPEAR-Crystal Ball	SLAC-PUB-4160 (1986)	Osterheld, A.	R	full	3.670 - 4.496
SPEAR-Crystal Ball	SLAC-PUB-5160 (1989)	Edwards, C.	R	full	5.0 - 7.4
SLAC-MARK-II	SLAC-219 (1979)	Schindler, R.H.	R	full	3.670 - 3.872
DORIS-DASP	Phys. Lett. **76B** (1978) 361	Brandelik, R.	R	b/m	3.6025 - 5.1950
DORIS-II-LENA	Z. Phys. **C15** (1982) 299	Niczyporuk, B.	R	full	7.440 - 9.415
* DORIS-II-ARGUS	Z. Phys. **C54** (1992) 13	Albrecht, H.	R	full	9.360
DORIS-II-Crystal Ball	Z. Phys. **C40** (1988) 49	Jakubowski, Z.	R	full	9.39 - 9.46
* DORIS-II-DHHM	Z. Phys. **C6** (1980) 125	Bock, P.	R	full	9.45 - 10.04
DORIS-II-DASP	Phys. Lett. **116B** (1982) 383	Albrecht, H.	R	full	9.51
VEPP-4-MD1	Z. Phys. **C70** (1996) 31	Blinov, A.E.	R	full	7.30 - 10.29
CESR-CUSB	Phys. Rev. Lett. **48** (1982) 906	Rice, E.	R	full	10.43 - 11.09
CESR-CLEO	Phys. Rev. **D29** (1984) 1285	Giles, R.	R	full	10.49
CESR-CLEO ††	Phys. Rev. Lett. **54** (1985) 381	Besson, D.	R	NO	10.60 - 11.20
CESR-CLEO II	Phys. Rev. **D57** (1998) 1350	Ammar, R.	R	full	10.52
DORIS/PETRA-PLUTO	Phys. Rep. **83** (1982) 151	Criegee, L.	R	full	3.6 - 30.8
* PETRA-TASSO	Z. Phys. **C22** (1984) 307	Althoff, M.	R	full	12.0 - 41.4
* PETRA-TASSO	Z. Phys. **C4** (1980) 87	Brandelik, R.	R	full	12.00 - 31.25
* PETRA-TASSO	Z. Phys. **C47** (1990) 187	Braunschweig, W.	R	full	14.03 - 43.70
PETRA-TASSO	Phys. Lett. **138B** (1984) 441	Althoff, M.	R	full	41.45 - 44.20
PETRA-JADE	Phys. Rep. **148** (1987) 67	Naroska, B.	R	full	12.00 - 46.47
PETRA-MARK-J	Phys. Rev. **D34** (1986) 681	Adeva, B.	R	full	12.00 - 46.47
* PETRA-MARK-J	Phys. Lett. **85B** (1979) 463	Barber, D.P.	R	full	31.57
* PETRA-MARK-J	Phys. Lett. **108B** (1982) 63	Barber, D.P.	R	full	34.85
PETRA-CELLO	Phys. Lett. **183B** (1987) 400	Behrend, H.J.	R	full	14.0 - 46.6
PEP-MAC	Phys. Rev. **D31** (1985) 1537	Fernandez, E.	R	full	29.0
* PEP-MARK-II	Phys. Rev. **D43** (1991) 34	von Zanthier, C.	R	full	29.0
* TRISTAN-AMY	Phys. Rev. **D42** (1990) 1339	Kumita, T.	R	full	50.0 - 61.4
* TRISTAN-TOPAZ	Phys. Lett. **234B** (1990) 525	Adachi, I.	R	full	50.0 - 61.4
* TRISTAN-TOPAZ	Phys. Lett. **347B** (1995) 171	Miyabayashi, K.	σ	full	57.77
* TRISTAN-TOPAZ	Phys. Lett. **304B** (1993) 373	Abe, T.	σ	full	57.37 - 59.84
* TRISTAN-VENUS	Phys. Lett. **198B** (1987) 570	Yoshida, H.	R	full	50.0 - 52.0
* TRISTAN-VENUS	Phys. Lett. **246B** (1990) 297	Abe, K.	R	full	63.6 - 64.0

† The measured cross section excludes all-neutral final states and not used in our analysis.

†† Data were not corrected for ISR and thus not used in our analysis.

All preliminary data are excluded. Data covering J/ψ, $\Psi(2S)$ and $\Upsilon(1S)..\Upsilon(4S)$ are also omitted as they are distorted by machine energy spread and cannot be directly used for the evaluation of $\Delta\alpha_{\rm QED}^{\rm had}(s)$ and $a_\mu(\rm had, LO)$.
Data from the SPEAR-DELCO experiment [Phys. Rev. Lett. **40** (1978) 671] are excluded as they were not corrected for the $\tau^+\tau^-$ contamination of the hadronic sample.